Stochastic Processes and their Applications 35 (1990) 181-212
North-Holland

EIGHTEENTH CONFERENCE ON STOCHASTIC PROCESSES AND THEIR APPLICATIONS

Madison, WI, USA, 25 June - 1 July 1989

Introduction

The Eighteenth Conference on Stochastic Processes and their Applications was held at the University of Wisconsin - Madison, USA, from 25 June to 1 July 1989. The conference was arranged under the auspices of the Committee for Conferences on Stochastic Processes of the ISI's Bernoulli Society for Mathematical Statistics and Probability and was held with the sponsorship and support of the following organizations: the University of Wisconsin - Madison Department of Mathematics, the National Science Foundation, the Air Force Office of Scientific Research, the Army Research Office, the National Security Agency and the Office of Naval Research.

The principal organizers of the conference were D. Griffeath (chair), J. Kuelbs and T. Kurtz. The Program Committee consisted of D. Griffeath (chair), A. de Acosta, M. Bramson, P. Glynn, F. Kelly, M. Pinsky and R. Williams.

The conference was attended by about 150 participants, including about 50 from outside the USA and Canada. The scientific program consisted of 15 invited lectures and 70 contributed papers. The following are the abstracts of the papers presented.

Abstracts[1]

1. Invited lectures

Large finite clusters in two-dimensional percolation

Kenneth S. Alexander, University of Southern California, Los Angeles, CA, USA*
Jennifer T. Chayes and Lincoln Chayes, University of California, Los Angeles, CA, USA

For independent bond percolation in two dimensions at density p above the critical point, some questions relating to large finite clusters are considered. The probabilities $P_{1-p}[0 \leftrightarrow x]$ of connection by a path of occupied bonds at density $1-p$ are used in constructing a natural norm $g_p(\cdot)$ on $\mathbb{R}^2$. It is shown that the probability $P_p[|C(0)| = N]$ that the cluster of the origin has finite size N behaves like $\exp(-R(p)N^{1/2})$ as $N \to \infty$, where $R(p)$ involves the correlation length, the percolation probability, and an isoperimetric constant: the minimum g_p-length of any curve

[1] An asterisk is attached to the name of the speaker in the case of a joint paper.

enclosing unit area. If $W(p)$ is the region enclosed by the corresponding shortest curve, then conditionally on $[|C(0)| = N]$, the cluster $C(0)$ of the origin closely approximates $W(p)$ in shape, with probability approaching one as $N \to \infty$.

The transient behavior of networks of queues

V. Anantharam, Cornell University, Ithaca, NY, USA

A closed Jackson network is a network of $\cdot/M/1$ queues, where service at each of the nodes is FCFS and customers leaving a node are routed to one of the other nodes according to a stochastic matrix, the routing decisions being independent of the service times. We will discuss the probabilistic evolution of such a network when there is a large number of customers circulating. In particular, we will prove the existence of a threshold phenomenon in the approach to stationarity. The estimates going into the proof also allow one to derive a useful rule of thumb for the buffer allocation problem for open networks of queues — a problem of considerable applied interest. This derivation will also be presented.

Statistical equilibrium and mixing for stochastic flows of diffeomorphisms

Peter H. Baxendale, University of Southern California, Los Angeles, CA, USA

Let $\{\xi_t : t \geq 0\}$ be a stochastic flow of diffeomorphisms of a compact manifold M. That is, $\{\xi_t : t \geq 0\}$ is a Diff(M) valued process with continuous sample paths, started at Id, with stationary left increments $\xi_t \xi_s^{-1}$ (for $0 \leq s \leq \infty$). Let $P(M)$ denote the space of Borel probability measures on M. Then $\mu_t = \mu \xi_t^{-1}$ ($\mu \in P(M)$) induces a Markov process $\{\mu_t : t \geq 0\}$ in $P(M)$.

If μ is the unit mass at x in M then $\{\mu_t : t \geq 0\}$ is naturally isomorphic to the one-point motion $\{\xi_t(x) : t \geq 0\}$ in M. More interestingly, if μ is stationary for the one-point motion then (using martingale convergence) μ_t converges in distribution as $t \to \infty$ to the so-called statistical equilibrium. Furthermore, in a large class of examples the statistical equilibrium consists almost surely of measures of Hausdorff dimension strictly between 0 and dim(M). (Results due to LeJan and Ledrappier and Young.)

In this talk we study the behavior as $t \to \infty$ of μ_t for arbitrary μ in $P(M)$. In particular we investigate when (and at what rate) $\mu_{1,t} - \mu_{2,t}$ converges to zero as $t \to \infty$ for distinct μ_1 and μ_2 in $P(M)$.

Non-intersection exponents for Brownian paths

Krzysztof Burdzy, University of Washington, Seattle, WA, USA

Let X and Y be independent n-dimensional Brownian motions, $n \geq 2$, $X(0) = 0$, $|Y(0)| = \varepsilon$, and let $p_n(\varepsilon) = P(X[0,1] \cap Y[0,1] = \emptyset)$. Asymptotic estimates (when $\varepsilon \to 0$) of $p_n(\varepsilon)$ and some related probabilities will be discussed. Applications to geometry of Brownian fractals will be given.

Sample path properties of a class of hypoelliptic diffusion processes

Mireille Chaleyat-Maurel, Université Paris VI, France

We study a simple class of hypoelliptic non-elliptic diffusion processes in $\mathbb{R}^3$. We give precise estimates for the Green function of the process and the capacity of small compact sets. The proofs rely on a study of the local behavior of the process, developed along the lines of Azencott, and on some bounds for the Green function which have been recently derived by analytic methods. Our estimates can be applied to various sample path properties, such as the limiting behavior of the volume of a small tubular neighborhood of the path, or the existence of multiple points. We emphasize the differences with the elliptic case. In particular, the volume of a tubular neighborhood of the path is shown to be smaller than for an elliptic diffusion.

Gibbs shape of a droplet

Roland Dobrushin, Institute of Problems in Information Transmission, Moscow, USSR

We consider the Gibbs distribution for a low-temperature two-dimensional Ising model with a periodic boundary condition in volume V. We introduce an additional condition that the total spin takes a fixed value $M(V)$. Supposing that $\lim M(V)/|V| = m$ if $|V|$ goes to infinity where $|m| > \frac{1}{2}$, we consider the asymptotic limit of the corresponding conditional distribution. It turns out that it converges to the situation when there is a droplet of one phase inside the other phase. This limit droplet has a fixed shape which is called the Wulff shape in statistical mechanics. This result was obtained by R. Kotezky, S. Shlosman and myself. In the talk many open questions connected with the generalization of this result will also be discussed.

Slab results for percolation and contact processes

Geoffrey Grimmett, University of Bristol, UK

Of the many major open problems for infinite particle systems and random media, some of the most accessible concern the behavior of processes at their critical points. The critical branching process dies out (barring the special case), but is the same true for percolation and/or the contact process, say? If this is so, then what about critical exponents...?

Related to the question about behavior at the critical point is the question of deciding to what degree a process in d dimensions may be 'approximated' by a process living on an essentially two-dimensional subset (there are special techniques for processes in two dimensions).

We arrive thus at the following matters:

Percolation: Is there percolation when the density of open bonds *equals* its critical value p_c? Is the limit of the critical probabilities of slabs equal to p_c?

Contact process: Does the contact process survive when the infection rate *equals* its critical value λ_c? What does the infected region in space-time look like, for general starting sets and infection rates?

Recent process in slab technology has led to sensible answers to almost all of these questions, and hence resolutions of several problems of interest for supercritical percolation and contact processes.

Load balancing for a random graph

B. Hajek, University of Illinois, Urbana, IL, USA

A set of M resource locations and a set of αM consumers are given. Each consumer requires a specified amount of resource and is constrained to obtain the resource from a specified subset of locations. The problem of assigning consumers to resources so as to balance the load among the resource locations as much as possible is considered. It is shown that there are assignments, termed uniformly most balanced assignments, that simultaneously minimize certain symmetric, separable, convex cost functions.

The distribution function of the load at a given location for a uniformly most balanced assignment is studied assuming that the set of locations each consumer can use is random. An asymptotic lower bound (conjectured to be asumptotically exact) on the distribution function is given for M tending to infinity, and an upper bound is given on the probable maximum load.

It is shown that there is typically a large set of resource locations that all have the maximum load, and that for large average loads the maximum load is near the average load.

Probabilistic methods in differential geometry

Pei Hsu, Northwestern University, Evanston, IL, USA

We will introduce briefly Riemannian Brownian motion and the minimal heat kernal on a Riemannian manifold and discuss how the Ricci curvature and sectional curvature affects the behavior of Brownian motion. We then show how Brownian motion can be used to prove some old and new results in differential geometry, including the existence of harmonic functions on Cartan–Hadamard manifolds, small time asymptotic behavior of the heat kernal, the stochastic completeness and Feller property of the heat semigroup, and heat kernel on noncomplete Riemannian manifolds.

A trajectorial construction of some measure-valued processes

Jean-Francois Le Gall, Université Paris VI, France

We give a trajectorial construction of some measure-valued Markov processes which have been studied recently by Dawson, Dynkin, Perkins and others. These processes, called superprocesses by Dynkin, were originally constructed as weak limits of systems of branching diffusions. Our approach uses the theory of excursions of linear Brownian motion, and the notion of the branching tree associated with a Brownian excursion. It yields a simple probabilistic description of the nonlinear semi-group that appears in the study of the measure-valued process. Another advantage of our method is as follows. If we interpret the value of the process at a given time t as a measure on a cloud of particles, our approach makes it possible to consider the induced measure on the set of paths, stopped at time t, of these particles. Some questions related to the genealogy of the particles can be treated in this way, and one may expect applications to sample path properties of the type investigated by Perkins.

Remarks on the Witten–Sander model

Peter March, Ohio State University, Columbus, OH, USA

The Witten–Sander model, or diffusion limited aggregation, is a simply stated cluster growth model on the plane lattice. We make an indirect statement about the growth of a scaled sequence of clusters of a similar type by a comparison with the ill-posed Hele–Shaw problem of fluid mechanics.

Direct and inverse problems for pulse reflection by randomly layered media

George Papanicolaou, Courant Institute, New York, USA

Consider a pulsed acoustic or elastic wave impinging on a material that is inhomogeneous, the earth's crust for example. The reflected signal is measured at the surface and it looks very noisy,

like a seismogram. What information about the medium can we extract from this signal? In reflection seismology this is a classic question that has received a great deal of attention over the last forty years. It has also received a lot of attention as a difficult problem in mathematics: the inverse scattering problem. We have begun a study of this question as a problem in stochastic processes. We model the reflecting material as a random medium, characterize all possible reflected signals as a particular class of nonstationary stochastic processes and then pose the recovery of material properties as a statistical estimation problem for this class of processes. We have also conducted extensive numerical simulations to test the theory; the estimation algorithms in particular.

On the behavior of some cellular automata

Roberto H. Schonmann, São Paulo University, Brazil

We study the behavior of some cellular automata related to Bootstrap Percolation. Each site of the Lattice $\mathbb{Z}^d$ is in state 0 or 1. Sites which are in state 1 remain so forever, while sites that are in state 0 may change to state 1 deterministically according to the states of their neighbors, following some specified rules. All the randomness is in the initial state.

A basic example is the one in which at each integer time a site which is in state 0 flips to state 1 if at least half of its neighbors are in state 1. We will show that in any dimension and for any initial i.i.d. distribution with positive density every site will eventually turn to state 1. We will then discuss how fast this happens. The methods used will provide some simple examples of techniques such as Peirls estimates and renormalization.

Applications of isoperimetric inequalities to sums of independent random variable

M. Talagrand, Université Paris VI, France

We introduce a new method to estimate the tails of sums of independent Banach space valued random variables. The method relies on two isoperimetric inequalities of independent interest, and yields sharp results.

On derivation of the Boltzmann equation from a deterministic motion of many particles

K. Uchiyama, Hiroshima University, Japan

We introduce a dynamical system of 'hard' squares (having velocities from a finite set) that seemingly corresponds to the Broadwell model of the B. eq. (Boltzmann equation) and discuss the Grad limit for it. For the hard-sphere model O. E. Lanford derived the B. eq. from the BBGKY hierarchy for short times by taking the Grad limit. For the present model an analogy is observed at the formal level: the first equation of the BBGKY hierarchy is reduced to the corresponding B. eq. (Broadwell model) by formally passing to the Grad limit. The actual limit, which can be shown to exist for short times, however does not solve the B. eq.

It is commonly understood that the B. eq. is valid under the hypothesis of Molecular Chaos, but both the precise meaning of the hypothesis and the way how it assures the B. eq. remain to be clarified. H. Grad pointed out that the chaos property for two particles is destroyed for post-collisional configurations and in order to assure the validity of the B. eq. it is needed only for those configurations in which particles are about to collide. Now is it plausible to suppose that if recollisions are negligible so that two particles that are about to collide with each other have rarely made interaction through their 'ancestorial' particles, then they must be uncorrelated?

Our model serves as a counter example to this supposition: the recollisions are negligible, still such two particles are correlated.

2. Contributed papers

On extremal theory for differentiable stationary processes

J.M.P. Albin, University of Lund, Sweden

Let $\{X(t)\}_{t\geqslant 0}$ be a stochastically differentiable strictly stationary stochastic process, and let $\{A_u\}_{u>0}$ be a decreasing family of rare subsets of $\mathbb{R}^m$, i.e. $P\{X(0)\in A_u\}\to 0$, as $u\to\infty$. We give a general method to derive an asymptotic expression for the probability $P\{X(t)\in A_u$ for some $0\leqslant t\leqslant h\}$ as $u\to\infty$. In particular, letting $A_u=(u,\infty)$ and $m=1$, the method suggests a way to study the 'usual' extremal problem.

Our approach, leaning more directly on the differentials of $X(t)$ than through their impact on the moments of the number of upcrossings, yields a clustering allowing extremal theory with conditions formulated in terms of f.d.d.'s of order three and lower, while the traditional upcrossing approach, which is not clustering allowing, needs conditions involving f.d.d.'s of order four.

Prediction for an autoregressive process with exponential white noise

Maria Antónia Amaral Turkman, University of Lisbon, Portugal

Autoregressive processes with non-Gaussian white noise are useful for modelling a wide range of phenomena which do not allow negative values. Such processes were studied by Gaver and Lewis (1980), Bell and Smigh (1986) among others. Andel (1988) considers the autoregressive model $X_t=\rho Y_{t-1}+Y_t$ where $0\leqslant\rho\leqslant 1$, Y_t are independent exponential random variables with mean value θ^{-1} and X_1 is exponential with mean value $(\theta(1-\rho))^{-1}$. This model was studied from a Bayesian point of view by Amaral Turkman (1989). In this paper we address the problem of predicting certain type of events which may be of interest in a practical situation modeled by this process.

References

M.A. Amaral Turkman, Bayesian analysis of an autoregressive process with exponential white noise (1989), submitted to Statistics.

J. Andel, On AR(1) processes with exponential white noise, Comm. Statist.—Theory Methods 17(5) (1988) 1481-1495.

C.B. Bell and E.P. Smith, Inference for non-negative autoregressive schemes, Comm. Statist.—Theory Methods 15(8) (1986) 2267-2293.

D.P. Gaver and P.A. Lewis, For order autoregressive gamma sequences and point of processes, Adv. Appl. Probab. 12 (1980) 727-745.

Brownian motion as the strong limit of the difference of two Poisson processes with an application to M/M/1 queues

Madjid A. Amir and Frank B. Knight, University of Illinois, Urbana, IL, USA*

We consider two independent Poisson processes $A_n(t)$ and $B_n(t)$ with rates λ_n and μ_n respectively defined on (Ω_n, F_n, P_n), for $n=1,2,3,\ldots$. Let $R_n(t)=(A_n(t)-B_n(t))/2^n$. $R_n(t)$ is then a birth and death process with state space the lattice $\{k/2^n, k = \pm 1, \pm 2, \pm 3, \ldots\}$, and birth rate (death rate resp.) $\lambda_n/(\lambda_n+\mu_n)$ ($\mu_n/(\lambda_n+\mu_n)$, resp.). We show the existence of a probability space $(\Omega_\infty, F_\infty, P_\infty)$ (projective limit space) such that $\Omega_\infty = \{\omega = (\omega_1, \omega_2, \omega_3, \ldots, \omega_n, \ldots), \omega_n \in \Omega_n\}$ and $P_\infty(S_n) = P_n(S_n)$ for $S_n \in F_n$ and $n = 1, 2, 3, \ldots$. We show that

$$P_\infty\left\{\lim_{n\to\infty} \sup_{t\in[0,1]} \left|\frac{A_n(t)-B_n(t)}{2^n} - W(t)\right| = 0\right\} = 1$$

where $W(t)$ is standard Brownian motion (Brownian with drift $-\ln(\alpha)$, resp.) when $\lambda_n = \mu_n = 2^{2^n}$ ($\lambda_n = 2^{2^n}$ and $\mu_n = 2^{2^n}\alpha^{1/2^n}$, resp.), where α is any positive real number. We thus construct a Brownian motion with and without drift as a strong limit of a sequence of birth and death processes. Note that the weak convergence in the space D will follow.

As many processes arising in applied probability can be modeled as functionals of the difference of two Poisson processes, this result has applications in areas such as queueing theory and storage theory. A natural application is to M/M/1 queues: Let $Q_n(t)$ be the queue length process of an M/M/1 queue with either set of rates above. Then

$$P_\infty\left\{\lim_{n\to\infty} \sup_{t\in[0,1]} \left|\frac{Q_n(t)}{2^n} - \bar{W}(t)\right| = 0\right\} = 1$$

where $\bar{W}(t) = W(t) - \inf_{s\le t} W(s)$ is reflected Brownian motion. This is a much stronger result than the usual weak convergence. Some rates of convergence have been derived.

Convergence to the normal distribution by graph methods

Florin Avram, Northeastern University, Boston, MA, USA

When establishing convergence to the normal distribution of sums of Wick products of a stationary sequence X_i by the method of cumulants, one is led to the study of the asymptotics of a certain type of deterministic sums associated with graphs. When the graph is a cycle, this 'graph-sum' is a product of Toeplitz matrices, and the asymptotics in this case was obtained by Szego (1958).

Two results concerning the asymptotics of these general 'graph-sums' have been obtained: An inequality which establishes their order of magnitude, as well as an exact limit theorem. The results show dependence on the bond matroid structure of the graph.

The main tool in establishing these resslts was showing that a Holder type inequality for multiple integrals of functions applied to linearly dependent arguments holds, provided certain rank conditions, known to physicists as the 'power counting' conditions, are satisfied.

Computing the cumulants by the diagram formulae and then applying our asymptotic results may reduce the establishing of rather complicated central limit theorems to easy graph problems.

The method assumes formally only stationarity of the underlying sequence X_i, and a certain analytical condition, which however is well understood only for Gaussian or linear X_i.

Aggregated Markov processes with negative exponential time interval omission

Frank Ball, University of Nottingham, England

We consider a time reversible, continuous time Markov chain on a finite state space. The state space is partitioned into two sets, termed open and closed, and it is only possible to observe whether the process is in an open or a closed state. Such aggregated Markov processes have found considerable application in the modelling and analysis of single channel records that occur in certain neurophysiological investigations. The aim of such single channel analysis is to draw inferences concerning the structure and transition rates of the underlying Markov process from the observed aggregated process.

A further problem with single channel analysis is that short sojourns in either the open or closed states are unlikely to be detected, a phenomenon known as time interval omission. We present a complete solution to the problem of time interval omission, in the situation when the length of minimal detectable sojourns follows a negative exponential distribution with mean μ^{-1}. We provide an asymptotic analysis as μ tends to infinity, which allows us to discuss the robustness of the above-mentioned structural inferences to time interval omission. We illustrate the theory with a numerical study of a model for the gating mechanism of the locust muscle glutamate receptor.

Local asymptotic mixed normality of log-likelihood based on stopping times and its rate of convergence

A.K. Basu and Debasis Bhattacharya, Calcutta University, India

Let Θ, the parameter space, be an open subset of $\mathbb{R}^k$, $k \geq 1$. For each $\theta \in \Theta$ let the r.v.'s X_m, $m = 1, 2, \ldots$, be defined on the probability space (Ω, A, P_θ) and take values in the space (S, ζ), where S is a Borel subset of a Euclidean space and ζ is the σ-field of Borel subsets of S. It is assumed that the joint probability law of any finite set of such r.v.'s $\{X_m, m \geq 1\}$ has some known functional form except the unknown parameter θ. For $h \in \mathbb{R}^k$ and a sequence of p.d. matrices $\delta_n = \delta_n^{k \times k}(\theta_0)$, set $\theta_n^* = \theta^* = \theta_0 + \delta_n^{-1} h$, where θ_0 is the true value of θ, as one value of θ. For each $n \geq 1$, let v_n be stopping times defined on the process, with some desirable properties. Let $\Lambda_m(\theta^*, \theta_0)$ be the log-likelihood ratio of the probability measure $P_{m\theta^*}$ w.r.t. the probability measure $P_{m\theta_0}$, where $P_{m\theta}$, is the restriction of P_θ over $A_m = \sigma\langle X_1, X_2, \ldots, X_m\rangle$. Replacing m by v_n in $\Lambda_m(\theta^*, \theta_0)$ we get the randomly stopped log-likelihood ratio, namely $\Lambda_{v_n}(\theta^*, \theta_0)$. The main purpose of this paper is to show that under certain regularity conditions the limiting distribution of $\Lambda_{v_n}(\theta^*, \theta_0)$ is locally asymptotically mixed normal. Two examples are also taken into account. First a Berry-Essén theorem is obtained under LAN condition. Then a rate of convergence is obtained under LAMN conditions.

Some probabilistic problems arising from an optimal-path algorithm

J. van den Berg, Centre for Mathematics and Computer Science, Amsterdam, The Netherlands

Consider a finite graph G. Suppose that with each edge e two types of cost, $c(e)$ and $d(e)$ are associated.

Let A and B be vertices of G. The path from A to B which minimizes cost c can be found by Dykstra's algorithm. The running time of this algorithm is polynomially bounded in the number of edges of G. If one wants to minimize cost c under an additional constraint on cost d, or, more generally, if one wants to minimize an increasing but non-linear function of the total cost c and d, this can be done by a generalised (multiple-labelling) Dykstra algorithm. The running time is now closely related to the number of efficient paths between two given vertices (a path is efficient if there is no other path of which both cost c and cost d are smaller). It is not difficult to construct an example where this grows exponentially with the number of edges. We take the costs $d(e)$ and $c(e)$ stochastic and study the average number of efficient paths. This leads to more general interesting probabilistic problems of a combinatorial nature.

An approximate martingale central limit theorem in Hilbert space

Michael S. Bingham, University of Hull, UK

Let H be a real separable Hilbert space with inner product $(\cdot,\cdot)$. By an S-operator we mean a nonnegative linear operator on H with finite trace. Suppose that we are given an adapted triangular array $\{S_{nj}, \mathscr{F}_{nj}: 0 \leq j \leq k_n, n > 1\}$ of H-valued random variables defined on the probability space $(\Omega, \mathscr{F}, P)$. Define $X_{nj} := S_{nj} - S_{n,j-1}$. Assume the following:

For every neighborhood N of 0 in H,

$$P(X_{nj} \notin N \text{ for some } j = 1, 2, \ldots, k_n) \to 0 \quad \text{as } n \to \infty.$$

There is a neighborhood M of 0 in H such that $X'_{nj} := 1_M(X_{nj})X_{nj}$ satisfy

$$\sum_{j=1}^{k_n} |E[(X'_{nj}, y)|\mathscr{F}_{n,j-1}]| \xrightarrow{P} 0 \quad \text{as } n \to \infty$$

and

$$\sum_{j=1}^{k_n} (X'_{nj}, y)^2 \xrightarrow{P} (Sy, y) \quad \text{as } n \to \infty$$

for every $y \in H$ where S is a random S-operator on H. Assume also that there is a fixed S-operator T on H such that

$$E\left(\sum_{j=1}^{k_n} |E[(X'_{nj}, y)|\mathscr{F}_{n,j-1}]|\right) \leq \sqrt{(Ty, y)} \quad \text{for all } y \in H$$

and

$$E\left[\sum_{j=1}^{k_n} (X'_{nj}, y)^2\right] \leq (Ty, y) \quad \text{for all } y \in H$$

both hold for all sufficiently large n. Finally, assume also that the filtrations are nested: i.e. $\mathscr{F}_{n,j} \subseteq \mathscr{F}_{n+1,j}$ for all n, j.

Then S_{nk_n} converges stably in law as $n \to \infty$ to the mixture of Gaussian distributions with characteristic function: $y \to E[\exp[-1/2(Sy, y)]]$, $y \in H$.

Stochastic models for the evolution of social behaviour based on the resource dispersion hypothesis

Paul Blackwell, University of Nottingham, England

The resource dispersion hypothesis suggests that heterogeneity of the distribution of food may sometimes create opportunities for animals to share territories, without any cooperation or social interaction. The evolutionary model presented here attempts to determine when the strategy of exploiting such an opportunity will be a successful one.

A juvenile forming a group with its parents may improve its chance of survival, it may cointribute to its parents' future reproductive success, and it may inherit its parents' territory on their death. The alternative strategy is for a juvenile to try to establish a territory in its own right, and breed immediately.

The success of each strategy is clearly density dependent. In addition, a distinction must be made between established adults and juveniles, and birth and death rates depend on the grouping of the population. We model the system as a Markov chain, with the state depending not only on the number of users of each strategy, but also on their organization into groups. When one strategy is rare the system can be approximated by a branching process. The global behaviour of the system can be investigated using numerical or simulation techniques.

Law of large numbers and central limit theorem for a stochastic model of a nonlinear chemical reaction with diffusion

Douglas Blount, University of Utah, Salt Lake City, UT, USA

A stochastic model of a nonlinear chemical reaction with diffusion is constructed by dividing the unit interval into N cells, allowing particles within each cell to reproduce as a nonlinear birth-death process, and coupling cells by diffusion with jump rate proportional to N^2. Cell numbers are divided by 1, a parameter proportional to the initial number of particles in a cell. The rescaled quantities are viewed as concentrations and the resulting space-time Markov process is compared to the corresponding deterministic model, a solution to a nonlinear partial differential equation. Letting 1 and N approach infinity in an appropriate manner allows one to prove a law of large numbers and central limit theorem. By using a new method the limits are shown to hold in the best possible state spaces, improving previous results of Kotelenez.

A fluctuations limit for scaled age distributions and weighted Sobolev spaces

A. Bose, Carleton University, Ottawa, Ont. Canada

It is shown that under the central limit scaling, the fluctuations of the space-time renormalized age distributions of particles (whose development is controlled by critical linear birth and death processes) around the law of large numbers limit converge in a Hilbert space (dual of a weighted Sobolev space containing the class of signed Radon measures with finite moment generating functionals) to a continuous Gaussian process satisfying a Langevin equation. So far, the space of rapidly decreasing functions has been considered to be the natural state space for the kind of limit theorem considered here. However, this space of rapidly decreasing function is not suitable in the present context and we are led to define a family of Sobolev spaces which are appropriate for our purpose.

Dependencies in branching processes

Per Broberg, Chalmers University of Technology, Göteborg, Sweden

In my talk I will survey some work on dependencies in branching processes, mainly my own, cf. the references.

A model allowing for reproductive dependencies between siblings will be presented, put in a wider context and related to other authors' works, including those of Jagers, Neveu and Mode.

References

Critical branching populations with reproductive Sibling correlations, Stochastic Process. Appl. 30 (1988) 133–147.

A note on the extinction probability of branching populations, Scand. J. Statist. 14 (1987) 125–129.

Limit theorems via large deviations

Wlodzimierz Bryc, University of Cincinnati, OH, USA

Limit theorems, which can be obtained from the Large Deviation Principle will be discussed. Some known and new results on Large Deviation Proofs of the Law of Large Numbers, Central Limit Theorem and Conditional Limit Theorems, each under appropriate additional assumptions, will be presented.

Trapezoidal Monte Carlo integration

Elias Masry, University of California at San Diego, La Jolla, CA, USA
Stamatis Cambanis, University of North Carolina, Chapel Hill, NC, USA*

The approximation of weighted integrals of random processes by the trapezoidal rule based on an ordered random sample is considered. For processes which are once mean-square continuously differentiable and for weight functions which are twice continuously differentiable, it is shown that the rate of convergence of the mean-square integral approximation error is precisely n^{-4}, and the asymptotic constant is also determined.

Markov branching processes with instantaneous immigration

Anyue Chen and Eric Renshaw, University of Edinburgh, UK*

Markov branching processes with instantaneous immigration (the conditional expectation of the sojourn time at extinction is zero), form a structure which represents a departure from general branching processes. In this paper we prove that if the sum of the immigration rates is finite then no such structure can exist; a necessary and sufficient condition for existence is the given for the case in which the sum is infinite. Study of the uniqueness problem shows that for honest processes the solution is unique.

Finite reversible nearest-particle systems in inhomogeneous and random environments

Dayue Chen, University of California, Los Angeles, CA, USA

In this paper we propose and study finite reversible nearest-particle systems in inhomogeneous and random environments. Using the Dirichlet principle and the ergodic theorem we prove that a finite reversible nearest-particle system in random environments survives if $E \log \lambda > 0$ and dies out if $E\lambda < 1$. Some discussion is provided to show both survival and extinction may happen when $E \log \lambda < 0$ and $E\lambda > 1$.

On some inequalities of local time

Chou Ching-Sung, National Central University, Chung-Li, Taiwan, R.O.C.

Let $(\Omega, F, (F_t)_{t\geq 0}, P)$ be a probability space satisfying the usual conditions. Let $L:(\Omega, F) \to \mathbb{R}_+$ be a positive random variable (not necessarily a (F_t)-stopping time). Let G^L_∞ be the σ-field generated by F_∞ and L. Consider

$$G_t^L = \{B \in G^L_\infty;\ \exists B_t \in F_t \text{ such that } B_t \cap (t > L) = B \cap (t < L)\};$$

then (G_t^L) satisfies the following properties:

(a) $F_t \subseteq G_t^L$, $t \geq 0$;
(b) (G_t^L) verifies the usual conditions;
(c) L is a (G_t^L)-stopping time.

Let $Z_t^L = E[I_{(L>t)} | F_t]$. $Z^L = (Z_t^L)$ is a potential of class (D). The decomposition of Doob–Meyer is $Z^L = M - A$. We put $I_L = \inf 0 \leqslant s < L Z_s^L$ and use Z to denote Z^L.

Our principal results are local time inequalities stopping at any time, which generalize results of Barlow-Yor.

Theorem. *Assume that* M *is continuous and* $\langle((1/z)\cdot M)^L\rangle_\infty$ *is a.s. finite. Then, there exists a constant* c_p *depending only on* p, $1\leq p<\infty$, *such that for every continuous* (F_t)*-local martingale* X, λ_t^a *the local time of* X, $a\in\mathbb{R}$, *we have*:

$$(1)\qquad E[\lambda_L^{*p}]\leq C_pE\left[\left(1+\log\frac{1}{I_L}\right)^{p/2}\langle X\rangle_L^{p/2}\right],\quad \lambda_t^*=\sup_{a\in\mathbb{R}}\lambda_t^a;$$

$$(2)\qquad E[\lambda_L^{*p}]\leq C_pE\left[\left(1+\log\frac{1}{I_L}\right)^{p}X_L^{*p}\right],\quad X_t^*=\sup_{s\leq t}|X_s|;$$

$$(3)\qquad E[X_L^{*p}]\leq C_pE\left[\left\{1+\left(\log\frac{1}{I_L}\right)^{p}\right\}\lambda_L^{*p}\right];$$

$$(4)\qquad E[\langle X\rangle_L^p]\leq C_pE\left[\left\{1+\left(\log\frac{1}{I_L}\right)^{p}\right\}\lambda_L^{*p}\right];$$

where C_p *can change from place to place.*

Upon a possibility of generation of random graphs

Moise Cocan, University of Braşov, Romania

The present paper studies a possibility of generation of random graphs by using the concept of Markov process.

We study the Markov process $(\xi(t))_{t\in\mathbb{N}}$ where $\xi(t)=G_t$, $t\in\mathbb{N}$, $G_t=(V_t,M_t)$, $V_t\neq\emptyset$, $|V_t|=n_t$, $|M_t|=m_t$, $n_t+m_t=t$, $t\in\mathbb{N}$, and $G_1=(V_1,\emptyset)$, $|V_1|=1$.

Let's presume that G_t is obtained from G_{t-1} by adding a new node or a new edge. It is clear that if the graph G_{t-1} is complete, G_t is obtained by adding a new node. The distribution of probability of the random variable G_t (G_t is a variable with graph values) is

$$\begin{bmatrix}(V_{t-1}\cup\{v\},M_{t-1}) & V_{t-1},M_{t-1}\cup\{m\}) \\ p & q\end{bmatrix}$$

where $p,q\geq 0$, $p+q=1$, $v\notin V_{t-1}$, $m\notin M_{t-1}$.

Different feature of graphs G_t ($t\in\mathbb{N}$) are studied and a certain algorithm is given for the generation of sample subgraphs of the graph G_t ($t\in\mathbb{N}$).

For the generation of graphs G_t ($t\in\mathbb{N}$), an algorithm is given which is able to select those that have certain features.

Considering graph G_t ($t\in\mathbb{N}$), t-fixed, as a structured population and the sample subgraphs as samples from this population, a study of possibilities of estimation of parameters that show the character of the considered graph is realised.

Hitting probabilities for spectrally positive Lévy processes

R.A. Doney, York University, Downsview, Ont., Canada

Let $\{Y(t),t\geq 0\}$ be a spectrally positive Lévy process, that is a process with stationary and independent increments which has no negative jumps, and let $\tau(x)$ denote the hitting probability of the singleton $\{x\}$, viz. $\tau(x)=\Pr\{Y(t)=x$ for some $0<t<\infty\}$. We give a formula in terms of the Lévy exponent of Y for the Laplace transform of $\tau(x)$, and use it to determine explicitly the value of $\tau(0)$ and $\lim x \to \infty\ \tau(x)$. In the case that $Y(t) = U(t) - t$, where U is a subordinator, or non-decreasing Lévy process, the first result confirms a recent conjecture of Mallows and Nair and the second extends a result of Nair, Shepp and Klass for the case that U is a Poisson process. We also give formulae for the Laplace transform of $E(e^{-\theta T(x)})$ and $E(e^{-\theta D(x)})$, where $T(x)$ is the time at which the first visit to x takes place, and $D(x)$ is the time between the first passage over x and $T(x)$.

Duality principle in doubly controlled processes of servicing machines with reserve replacement

Jewgeni Dshalalow, Florida Institute of Technology, Melbourne, FL, USA

The classical system of servicing $m+1$ unreliable machines by a single repairman was studied by Takács (1962). In the present modification of this model, the author allows w additional machines to be placed on reserve substituting any failed one. In addition, a comprehensive control over the failure rates and repair times maintains the whole system. The author introduces two models, with and without idle periods of the repairman, and he defines and applies a duality principle to find an explicit relation between the servicing processes in both models. The author further develops semi-regenerative techniques (invented earlier, 1985, 1989a) and applies them to the servicing process in the model without idle periods which seem to be simpler analytically. Then, the duality relation allows to extend the results obtained for a more attractive system with idle period.

References

J. Dshalalow, On a multi-server queue with finite waiting room and controlled input, Adv. Appl. Probab. 17 (1985) 408-423.

J. Dshalalow, On a duality principle in processes of servicing machines with double control, J. Appl. Math. Simul. 1(3) (1988) 245-252.

J. Dshalalow, Multichannel queueing systems with infinite waiting room and stochastic control, J. Appl. Probab. 26 (1989a).

J. Dshalalow, Doubly controlled processes of servicing machines with reserve replacement (1989b), submitted to Stochastic Process. Appl.

L. Takács, Introduction to the Theory of Queues (Oxford Univ. Press, Oxford, 1962).

The distribution of the frequencies of age-ordered alleles in a diffusion model

S.N. Ethier, University of Utah, Salt Lake City, UT, USA

We prove that the frequencies of age-ordered alleles in the stationary infinitely-many-neutral-alleles diffusion model are distributed as

$$X_1, (1-X_1)X_2, (1-X_1)(1-X_2)X_3, \ldots,$$

where $X_1, X_2, X_3, \ldots$, are independent beta$(1, \theta)$ random variables, θ being twice the mutation intensity; that is, the frequencies of age-ordered alleles have the so-called Griffiths-Engen-McCloskey, or GEM, distribution. In fact, two proofs are given, the first involving reversibility and the size-biased Poisson-Dirichlet distribution, and the second relying on a result of Donnelly and Tavaré relating their age-ordered sampling formula to the GEM distribution.

Cyclic cellular automata and related processes

Robert Fisch, University of North Carolina at Charlotte, NC, USA

A cyclic cellular automaton is a descrete-time process defined on the state space $\{0, 1, \ldots, N-1\}^{\mathbb{Z}^d}$. Initially, each lattice site of $\mathbb{Z}^d$ is randomly assigned one of the values from $\{0, 1, \ldots, N-1\}$, which are called *colors*. The N colors are arranged in a cyclic heirarchy; we say that color i can *eat* color j if $i-j=1 \pmod N$. Given an initial configuration, the evolution proceeds deterministically. At each time step, each site x looks at the color of each of its neighbors. If the color of any neighbor can eat the color of x, then it does so, meaning that x changes its color to the eating color. All sites update synchronously in this manner at each time step.

Cyclic cellular automata have properties akin to cyclic particle systems, introduced by Bramson and Griffeath, which are continuous time Markov processes on $\{0, 1, \ldots, N-1\}^{\mathbb{Z}^d}$ with a stochastic evolution rather than a deterministic one. Some empirical observations of the two cyclic systems in two dimensions will be presented as qualitative comparisons. Then, the focus will shift to the one-dimensional cyclic cellular automaton to present some results concerning clustering. Finally, we shall discuss the relation of this clustering result to other theorems about one-dimensional systems of particles with various interaction mechanisms.

Regression models with increasing number of unknown parameters

A.G. Gadžiev, Azerbaijanian Academy of Sciences, Baku, USSR

We consider the regression model

$$y_i = \eta(x_i, \theta) + \mathscr{E}_i, \quad i = 1, \ldots, N,$$

where $\eta(x_i, \theta)$ is a some nonlinear function, $\theta = (\theta_1, \ldots, \theta_m)^{\mathrm{T}}$ is the vector of unknown parameters. Assume

$$E\mathscr{E}_i = 0 \text{ and } E\mathscr{E}_i^2 = \sigma_i^2 \text{ are unknown and } \sigma_i^2 \leq \sigma_0^2. \tag{1}$$

The number m of unknown parameters depends on N and

$$m/N \to 0, \quad N \to \infty. \tag{2}$$

Let $\hat{\theta}$ denote a least squared estimator,

$$\hat{f}_{ij} = \frac{\partial \eta(x_j, \hat{\theta})}{\partial \theta}, \qquad \hat{F} = (\hat{f}_{ij}), \qquad \hat{y} = \hat{F} \cdot \hat{\theta}, \qquad I_{k,l}(\hat{\theta}) = (\Delta_{ij}),$$

$$\Delta_{ij} = \delta_{ij} \cdot \frac{\partial \eta(x_i, \hat{\theta})}{\partial \hat{\theta}_k} \cdot \frac{\partial \eta(x_j, \hat{\theta}}{\partial \hat{\theta}_l}.$$

$0 < \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_m$ are eigenvalues of the matrix $(\hat{F}^{\mathrm{T}}\hat{F}/N)^{-1}$, $C_N = NE(\hat{\theta}-\theta)(\hat{\theta}-\theta)^{\mathrm{T}}$ is the covariated matrix of the vector $\sqrt{N}(\hat{\theta}-\theta)$, C_{kl} are the elements of the matrix C_N,

$$a_{kl} = (\hat{y}-y)^{\mathrm{T}} I_{k,l}(\hat{\theta})(\hat{y}-y), \quad A = (a_{kl}), \qquad k, l = 1, \ldots, m,$$

and $\hat{C}_{kl}$ is the element (k, l) of the matrix $\hat{C}_N = (\hat{F}^{\mathrm{T}}\hat{F}/N)^{-1} A (\hat{F}^{\mathrm{T}}\hat{F}/N)^{-1}$. Let

$$m/N \cdot \lambda_1 \to 0 \quad \text{for } N \to \infty, \tag{3}$$

$$m \cdot \sqrt{m}/N \cdot \lambda_1 \to 0 \quad \text{for } N \to \infty. \tag{4}$$

Theorem. *If conditions* (1)–(3) *are true then*

$$(\hat{\theta} - \theta) \xrightarrow{\mathrm{P}} 0, \quad N \to \infty. \tag{5}$$

If condition (4) *is also true then*

$$E(\hat{C}_{kl} - C_{kl}) \to 0, \quad (\hat{C}_{kl} - C_{kl}) \xrightarrow{\mathrm{P}} 0, \qquad \text{for } N \to \infty. \tag{6}$$

If $m \leq c < \infty$ and $\eta(x_i, \theta) = \sum_{j=1}^{m} \theta_j \cdot \varphi_j(x_i)$, i.e., we have linear regression, then:

Corollary. *If $1/\lambda_1 \to 0$ as $N \to \infty$ then (5) and (6) hold.*

The matrix C_N can be used for the construction of a confidence band for the function $\eta(x, \theta)$. For example if $\mathscr{E}_i$ are Gaussian then the quadratic form

$$(\hat{\theta} - \theta)^{\mathrm{T}} \hat{C}_N^{-1} (\hat{\theta} - \theta)$$

has chi-squared distribution and it is easy to construct a confidence band to the function $\eta(x, \theta)$, for example according to the approach suggested in the reference.

Reference

A.G. Gadžiev, Construction a confidence region . . . , Preprint NO347-2809, Chalmers Univ., Göteborg (1984).

Uniqueness of viscosity solutions (V.S.) of Hamilton-Jacobi-Bellman (HJB) equations for controlled diffusions on finite-dimensional Riemannian manifolds with boundary

Giuliano Galeazzi, Via Gaetano Ciarrocchi, 15-CAP 00151, Rome, Italy

The notion of V.S. can be extended (talk at 17th Conference) to HJB equations on Riemannian manifolds M's:

$$G(p, V) = \inf_{a \in C} \{A^a(p, a_0) V(p) + f(p, a_0)\} = 0,$$

$$V(p) = 0 \quad \text{on } M.$$

(C = 'control actions,' A^a = 'controlled generat.') Uniqueness is now proven by semigroup arguments. A class of 2nd order elliptic operators which obey the Positive Maximum Principle has the property that (P_t), the C_0-diffusion semigroup associated with the given operator 'A,' maps any Borel and bounded g into $P_t g \in C^2(M)$ (due to R. Azencott). As V.S. are $C^0(M)$ and M is supposed compact, this shall hold for the difference of any 'two' V.S.: $P_t(V_1 - V_2) \in C^2(M)$. For V a V.S. and $p \in M$ there are sub-spaces $\hat{D}^+ V(p)$ and $\hat{D}^- V(p)$ of $C^\infty(B_p)$ (B_p is a geodesic ball cente-red at p) whose elements have, respectively, $G(p, H) \leq 0$ and $G(p, H) \geq 0$. Suppose there exists a dense subset $\hat{M}$ of M at whose points p_k, $\hat{D}^+ V(p_k) \neq \emptyset$ and $\hat{D}^- V(p_k) \neq \emptyset$ for any V.S. V. If there is a controlled generator $A^{\bar{a}}$ that is 'of Azencott's type' and such that for 'any' V.S. at each p_k there exists $H \in \hat{D}^+ V(p_k)$ that has $A^{\bar{a}}(p_k) H + f(p_k, a_0) \leq 0$, then picking just these H_1 for V_1 and any H_2 in $\hat{D}^- V_2(p_k)$: $A^{\bar{a}}(P_t(V_1 - V_2)) \leq A^{\bar{a}}(H_1 - H_2) \leq 0$ at p_k and t small enough. By density, $A^{\bar{a}}(P_t(V_1 - V_2)) \leq 0$ for all p's. The Classical Maximum Principle implies $P_t(V_1 - V_2) \geq 0$ on M. Sending $t \to 0^+$, $V_1 - V_2 \geq 0$. Finally, as the argument is symmetric w.r.t. V_1 and V_2, the converse inequality obtains too.

A geometric rate or convergence for the Maxwell gas with an angular cut-off

Brigitte Chauvin, Université Paris VI, France
Gaston Giroux, Université de Sherbrooke, Que., Canada*

We consider the measure-valued time-dependent solution to the Boltzmann equation for a Maxwell gas with an angular cut-off. For this solution, we show that, for any initial law having all its moments finite up to some integer, convergence to the equilibrium along good test functions occurs at a geometric rate.

Our proof is based on a representation of an underlying process involving an exponential age-dependent binary branching process. Some careful computation of conditional expectation is also needed.

This gives an answer to an old problem; see for instance the references.

References

H.P. McKean, Speed of approach to equilibrium for Kac's caricature of a Maxwellian gas, Arch. Rat. Mech. Anal. 21 (1966) 343-367.

L. Arkeryd, Stability in L^1 for the space homogeneous Boltzmann equation, Tech. Rept., University of Göteborg (Sweden, 1988).

Estimating return rates in some economic models

Victor Goodman, R.P.I., Troy, NY, USA

The price of an asset is assumed to satisfy an Itô stochastic differential equation over a time period where the solution remains positive. The return rate is defined to be the drift coefficient divided by the asset price.

A positive constant return rate is shown to produce positively correlated increments of the asset process. The problem of estimating a constant return rate is discussed. It is shown that for certain diffusion coefficients, the maximum likelihood estimation converges exponentially fast to the return rate.

Nonlinear limit for a system of diffusing particles which alternate between two states

Carl Graham, CMAP, Ecole Polytechnique, Palaiseay, France

We consider a system of interacting particles, each evolving alternately under two different diffusion dynamics in $\mathbb{R}^d$. For example, these particles could be alternatively in a gaseous and in a liquid phase, or adsorbed and desorbed by a medium. A special case is McKean's 2-speed model for the Boltzmann equation.

We study the corresponding nonlinear McKean–Vlasov diffusion, where a particle sees the medium as a function of its law. This can be considered as the statistical mechanics weak-interaction limit for the system, through a propagation or chaos result.

This is a probabilistic model for a chromatography tube, where molecules in a gaseous phase pushed by a flow of neutral gas are adsorbed and desorbed by a liquid phase in the tube. The time taken by the molecules to get through the tube is measured, and the device can be used to identify a substance, separate the components of a mixture, and so on.

Nonlinear problems may often be dealt with by contraction methods. This is straightforward in a stochastic differential equation setting, or for a pure-jump process whose generator integral form allows to work directly on the martingale problem. The problems here arise from the nonexistence of a good strong pathwise representation and from the integro-differential generator.

Our idea here is to construct processes having the appropriate laws, choosing the joint law so that the sample paths are as close as possible, and then use stochastic calculus. This is automatically done for strong stochastic differential equations by taking the same driving terms. Here we use time-change first, in order to get to a situation where strong pathwise contraction techniques apply. This is a rather new technique, developed in a previous joint paper with Professor Métivier. Thus, we obtain existence and uniqueness for the nonlinear process. Then we prove propagation of chaos.

Max infinitely divisible and max stable sample continuous processes

Evarist Giné, The City University of New York, USA, and Texas A&M University, College Station, TX, USA
Marjorie G. Hahn, Tufts University, Medford, MA, USA*
Pirooz Vatan, Strategic Automation Inc., Cambridge, MA, USA

Conditions for a process ξ on a compact metric space S to be simultaneously max infinitely divisible and sample continuous are obtained. Although they fall short of a complete characterization of such processes, these conditions yield complete descriptions of the sample continuous non-degenerate max stable processes on S and of the infinitely divisible non-void random compact subsets of a Banach space under the operation of convex hull of unions.

Correlation theory of almost periodically correlated processes

Harry L. Hurd, University of North Carolina, Chapel Hill, NC, USA

A second order stochastic process X is called almost periodically correlated (PC) in the sense of Gladyshev if its mean function $m(t)$ and covariance $R(t+\tau, t)$ are almost periodic functions of t for every τ. We show that the mean uniformly almost periodic processes discussed by Kawata are also almost PC in the sense of Gladyshev. If X is almost PC, then for each fixed τ function $R(t+\tau, t)$ has the Fourier series representation

$$R(t+\tau, t) \sim \sum_{\lambda \in \Lambda_\tau} a(\lambda, \tau) \exp(\mathrm{i}\lambda t) \tag{1}$$

and

$$a(\lambda, \tau) = \lim_{A\to\infty} \frac{1}{2A} \int_{-A+c}^{A+c} R(t+\tau, t) \exp(-\mathrm{i}\lambda t)\, \mathrm{d}t \tag{2}$$

exists for every λ and τ, independently of the constant c. Assuming only that $a(\lambda, \tau)$ exists in this sense for every λ and τ, we show $a(\lambda, \tau)$ is a Fourier transform

$$a(\lambda, \tau) = \int_{\mathbb{R}} \exp(\mathrm{i}\gamma\tau) r_\lambda(\mathrm{d}\gamma) \tag{3}$$

if and only if $a(0, \tau)$ is continuous at $\tau = 0$; under this same condition, the set $\Lambda = \{\lambda\colon a(\lambda, \tau) \neq 0$ for some $\tau\}$ is countable. We show that a strongly harmonizable process is almost PC if and only if its spectral support is contained in $\{(\gamma_1, \gamma_2)\colon \gamma_2 = \gamma_1 - \lambda_k, \lambda_k \in \Lambda\}$, a countable set of straight lines parallel to and including the main diagonal; further, one may identify the spectral measure on the kth line with the measure r_{λ_k} appearing in (5). Finally we observe that almost PC processes are asymptotically stationary and give conditions under which strongly harmonizable almost PC processes may be made stationary by an independent random time shift.

On the exchangeability of differentiation and stochastic integration with applications to stochastic PDE's

Jing-Lin Jiang, Wayne State University, Detroit, MI, USA

In this paper, we establish a result on the exchangeability of differentiation and stochastic integration, namely,

$$\frac{\partial}{\partial x} \int_0^T f(s, x, \omega)\, \mathrm{d}W_s = \int_0^T \frac{\partial}{\partial x} f(s, x, \omega)\, \mathrm{d}W_s$$

under some condition on f. A similar proposition is also true for higher-order derivatives. The results will be applied to the following two types of stochastic partial differential equations to yield classic solutions

$$\begin{aligned} &\frac{\partial U}{\partial t} = LLU + [f(t, x, \omega), W_t] && \text{in } D, \\ &U(0, x, \omega) = \phi(x, \omega) && \text{in } D, \\ &U(t, x, \omega) = 0 && \text{on } \partial D; \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial U}{\partial t} = LU + [hU + c \cdot \nabla U, W_t] && \text{in } \mathbb{R}^n, \\ &U(0, x, \omega) = \phi(x, \omega) && \text{in } \mathbb{R}^n; \end{aligned} \tag{2}$$

where L is a second order strong elliptic operator, $(\Omega, \mathcal{F}, P, \mathcal{F}_t, W_t)$ is an $\mathbb{R}^d$-valued standard Weiner process, D is a bounded domain in $\mathbb{R}^n$, $f:[0, T] \times d \times \Omega \to \mathbb{R}^d$, $h:[0, T] \times \mathbb{R}^n \to \mathbb{R}^d$, $c:[0, T] \times \mathbb{R}^n \to \mathbb{R}^{d \times n}$, $[\cdot, \cdot]$ denotes the scalar product in $\mathbb{R}^d$. An example of (1) is the heat equation ($L = \Delta$, the Laplacian) with the heat source $f(t, x, \omega) W_t$ of white-noise type. (2) arises from the nonlinear filtering problem of diffusion processes.

Random time change and integral representations for marked stopping times

Olav Kallenberg, Auburn University, AL, USA

The distributions of all random triples (τ, u, η), such that (τ, u) is a marked stopping time with compensator η, form a convex set M, whose extreme points are the distributions obtained when the filtration is the one induced by (τ, u). Moreover, every element of M has a unique integral representation as an average over extreme points.

Using the Doléans exponential process associated with η, one may define a certain discounted version ζ of the compensator, and it is shown that if T is a predictable mapping into $[0, 1]$ such that ζT^{-1} is bounded by Lebesgue measure, then $T_{\tau,u}$ is uniformly distributed on $[0, 1]$. The proof of this uses the peculiar fact that $EV_{\tau,u} = 0$ for any predictable process V satisfying a certain moment condition.

By iterating the mentioned time change result, one obtains a reduction of an arbitrary simple point process ξ to Poisson, through a predictable mapping based on the compensator η, together with suitable independent randomizations, needed to resolve the possible discontinuities of η at the atoms of ξ. Mention will also be made of a very general version of Knight's and Meyer's time change reduction of orthogonal continuous local martingales or quasi-leftcontinuous point processes to independent Brownian motions or Poisson processes, respectively.

Semimartingales forced onto a manifold by a large drift

G.S. Katzenberger, University of Wisconsin, Madison, WI, USA

We consider a sequence of $\mathbb{R}^d$-valued semimartingales

$$X_n(t) = Z_n(t) + \int_0^t F(X_n(s-))\, \mathrm{d}A_n(s),$$

where $\{Z_n\}$ is a 'well-behaved' sequence of semimartingales, F is a vector field whose deterministic flow has an asymptotically stable manifold of fixed points Γ and A_n is a nondecreasing process that asymptotically puts infinite mass on every interval. This situation occurs frequently in deriving diffusion approximations for certain genetics problems, branching processes and stochastic approximation problems. Intuitively, if X_n starts close to Γ then the drift term forces X_n to stay close to Γ and the limiting process (if there is one) must actually stay on Γ, hence is typically lower dimensional than the original X_n's. We give conditions under which $\{X_n\}$ is relatively compact in the *Skorohod topology* and obtain a stochastic differential equation for the limiting process(es).

A viscosity algorithm for the scalar conservation law

Andrzej Korzeniowski, University of Texas at Arlington, TX, USA

Consider a parabolic equation

$$\frac{\partial u_\sigma}{\partial t}=\tfrac{1}{2}\sigma^2\frac{\partial^2 u_\sigma}{\partial x^2}+B(u_\sigma)\frac{\partial u_\sigma}{\partial x},\quad u_\sigma(0,x)=g(x), \tag{1}$$

which reduces to a nonlinear hyperbolic equation known as the conservation law, when the viscosity σ is 0. Unlike the standard numerical schemes, which approximate differential operators by difference operators, we devise an algorithm for solving (1) that utilizes the Feynmann-Kac representation of the solution u_σ. It is well known that as $\sigma\to 0$ then $u_\sigma\to u$ which is a solution to

$$\frac{\partial u}{\partial t}=B(u)\frac{\partial u}{\partial x},\quad u(0,x)=0. \tag{2}$$

We compare our method with the Glimm's scheme, developed for (2), by means of numerical calculations for a concrete example. Our scheme is deterministic as opposed to Glimm's scheme, which requires a simulation of uniform random variables, and as such suffers no random number generation error.

The heavy traffic approximation for closed exponential queueing network

E.V. Krichagina, Institute of Control Sciences, Moscow, USSR

This report presents limit theorems for queue length processes associated with a general closed queuing network. The first theorem states that normalized sequences of queue length processes converge in probability to a deterministic function as the number of customers in the network goes to infinity. The limit function describes a unique solution of a certain differential inclusion. The second limit theorem establishes weak convergence of normalized deviations of the queue processes from a deterministic limit to a diffusion process with values in some convex region and instantaneous reflections on the boundaries.

Local times for superdiffusions

Steve Krone, University of Massachusetts, Amherst, MA, USA

We study local times for a class of measure-valued Markov processes known as superdiffusions. These processes arise as high-density limits of systems of independent branching $\mathbb{R}^d$-valued diffusions. As for some ordinary Markov processes, superdiffusions (when $d=1,2,3$) have local times. We will present some regularity results for these local times (e.g., joint continuity and Hölder condition) and implications about the behaviour of the superdiffusion (e.g., information about the Hausdorff dimension of the 'level sets').

Measuring the long-term dependence of some stationary stable processes

Joshua Levy, University of Lowell, MA, USA

To estimate the long-term dependence of a stationary stable process, $Y=\{Y(t)\}$, $-\infty<t<\infty$, we can evaluate the function $r:\mathbb{R}\to\mathbb{C}$ given by

$$r(t):=r_Y(\Theta_1,\Theta_2;t)=E\exp\{i(\Theta_1 Y(t)+\Theta_2 Y(0))\}\\ -(E\exp\{i\Theta_1 Y(t)\})(E\exp\{i\Theta_2 Y(0)\})$$

as $|t|\to\infty$ for all non-zero real numbers Θ_1 and Θ_2. We will study the case when Y is the increment process of either *linear fractional Lévy motion* (LFLM) or *log-fractional Lévy motion.* LFLM is an α-stable H-self-similar process defined for $0<\alpha\leqslant 2$, $0<H<1$, and $H\neq 1/\alpha'$ while log-fractional Lévy motion, which is defined only when $1<\alpha<2$, is α-stable and $(1/\alpha)$-self-similar.

One of the results for LFLM states that $r(t)\sim A|t|^{\alpha H-\alpha}$, $A=A(\alpha,H,\Theta_1,\Theta_2)$, if $1<\alpha<2$ and $1-1/(\alpha(\alpha-1))<H<1$. The current research extends earlier work of Austrauskas (1984, 1987) who studied the process *fractional Lévy motion*, which is one of the symmetric LFLM's. The constant A is important because there are α-stable H-self-similar processes, $Z=\{Z_{\alpha,H}(t)\}$, with $1<\alpha<2$ and $1-1/(\alpha(\alpha-1))<H<1$, that have stationary increments and that are generated by renewal sequences for which there exist $\Theta_1=\Theta_1(\alpha,H)$ and $\Theta_2=\Theta_2(\alpha,H)$ such that $r_Z(t)\sim A'|t|^{\alpha H-\alpha}$, but $A'\neq A$. As a consequence, Z cannot be any LFLM.

Brownian motion on nested fractals

Tom Lindstrøm, University of Oslo, Norway

The purpose of this paper is to construct Brownian motion on a reasonably general class of self-similar fractals. To this end, I introduce an axiomatically defined class of 'nested fractals', which satisfy certain symmetry and connectivity conditions, and which also are (in the physicists' terminology) finitely ramified. On each one of these nested fractals, a Brownian motion is constructed and shown to be a strong Markov process with continuous paths. If the Laplacian Δ on the fractal is defined as the infinitesimal generator of the Brownian motion, and $n(\alpha)$ denotes the number of eigenvalues of $-\Delta$ less than α, I prove that

$$n(\alpha)\sim\alpha^{d\log\nu/\log\lambda}\quad\text{as }\alpha\to\infty,$$

where d is the Hausdorff dimension of the fractal, and ν and λ are two parameters describing its self-similarity structure. In general, $\log\nu/\log\lambda\neq\frac{1}{2}$ and hence Weyl's conjecture can only hold for fractals in a modified form.

Using renewal theory to investigate self-avoiding walks

Neal Madras, York University, Downsview, Ont., Canada

It is possible to decompose some self-avoiding walks in a manner which translates questions about enumeration into questions about probabilities of renewal processes. (See, for example, H. Kesten, J. Math. Phys. 4, 1963, 960; or J.T. Chayes and L. Chayes, Comm. Math. Phys. 105, 1986, 221). We review this technique and show how probabilistic reasoning can be used to derive a rigorous (but probably nonoptimal) upper bound on the critical exponent for the number of self-avoiding polygons (also called self-avoiding loops).

Exponential stability for stochastic differential equations with respect to semimartingales

Xuerong Mao, University of Warwick, Coventry, England

Let $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ be a complete probability space with the filtration $\{\mathscr{F}_t\}$ satisfying the usual conditions, i.e., which is right continuous and contains all P-null sets. Consider the following stochastic differential equation with respect to semimartingales

$$\mathrm{d}X(t) = AX(t)\,\mathrm{d}\mu(t) + G(X(t), t)\,\mathrm{d}M(t), \quad t \geq t_0\ (\geq 0), \tag{1}$$

$$X(t_0) = x_0,$$

where $X = (X_1, \ldots, X_n)^{\mathrm{T}}$, A is a constant $n \times n$ matrix, μ is a continuous $\{\mathscr{F}_t\}$-adapted nondecreasing process, $M = (M_1, \ldots, M_m)^{\mathrm{T}}$ is an m-dimensional continuous local martingale with $M(0) = 0$, x_0 is an n-dimensional $\mathscr{F}_{t_0}$-measurable vector, and $G : \mathbb{R}^n \times \mathbb{R}_+ \times \Omega \to \mathbb{R}^{n\times m}$ is sufficiently smooth for the existence and uniqueness of the solution. Denote by $X(t, t_0, x_0)$ the solution of eq. (1).

Eq. (1) might be regarded as a stochastic perturbed system of

$$\mathrm{d}X(t) = AX(t)\,\mathrm{d}\mu(t). \tag{2}$$

Suppose eq. (2) is exponentially stable almost surely. Under what conditions is eq. (1) still exponentially stable almost surely?

Throughout this paper we will assume there exist $\{\mathscr{F}_t\}$-adapted process $K_{ij}(\cdot)$, $i, j = 1, 2, \ldots, m$, such that

$$\langle M_i, M_j\rangle(t) = \int_0^t K_{ij}(s)\,\mathrm{d}\mu(s), \quad t \geq 0. \tag{3}$$

Define $K := (K_{ij})_{m\times m}$. In this paper we prove the following main result.

Theorem. *Let λ_0 be the maximum of the real parts of all eigenvalues of $-A$. Suppose there exist positive constants α_1, α_2, nonnegative constants $\rho_1, \rho_2, \beta_1, \beta_2$, and polynomials $p_1(t), p_2(t)$ such that*

$$\|G(x, t)\|^2 \leq p_1(t)\,\mathrm{e}^{(-2\alpha_2\lambda_0+\rho_1)t} \quad a.s., \tag{4}$$

$$\|K(t)\| \leq p_2(t)\,\mathrm{e}^{\rho_2 t} \quad a.s., \tag{5}$$

$$\alpha_1 t - \beta_1 \leq \mu(t) \leq \alpha_2 t + \beta_2 \quad a.s. \tag{6}$$

Furthermore, assume

$$\limsup_{t\to\infty} \frac{1}{t} \log\|\mathrm{e}^{At}\|^2 \leq -\lambda \tag{7}$$

where λ is a positive constant. Then

$$\limsup_{t\to\infty} \frac{1}{t} \log\|X(t, t_0, x_0)\|^2 \leq -\alpha_1\lambda + \frac{\alpha_2}{\alpha_1}(\rho_1 + \rho_1) \quad a.s. \tag{8}$$

for all $t_0 \geq 0$ and $\mathscr{F}_{t_0}$-measurable vector x_0.

Steady state analysis for a series queueing network with blocking

Alain Jean-Marie, INRIA - Sophia Antipolis, Valbonne, France
William A. Massey, AT&T Bell Laboratories, Murray Hill, NJ, USA*

Finding the limiting equilibrium distribution for the finite state Markov chain associated with a queueing network that has finite buffers is a notoriously difficult problem. In this paper, we obtain an exact solution and new results for the equilibrium behavior of a class of series queueing networks with blocking. Each node has capacity one or buffer size zero. These systems arise in the modeling of data communication links or manufacturing assembly lines. A previous paper by the second author, used combinatorial methods to analyze such a network with the single Poisson input rate equal to the processing rate of all the N exponentially distributed service times. We generalize this model, by allowing the Poisson input rate λ to differ from the processing rates for each node, all of which are normalized to equal one. Aggregate mean value quantities for this system, such as the throughput rate, viewing this model as a truncated version of an asymmetric exclusion process. We extend these results to give a closed form solution for the complete joint queue length distribution, as well as the marginal distribution for any subset of the nodes.

These exact formulas are achieved by obtaining a recursion formula for the probabilities when suitably normalized. This leads to a simple expression for the grand partition function of the family of networks for fixed λ, indexed by N. We also use this generating function to develop simple estimates of these marginal probabilities. Specifically, we obtain for large N, simple asymptotic formulas for the marginal probabilities of a given node at the beginning, middle, and end of the network. Finally, we obtain an exact formula having the flavor of Little's law, that relates the throughput of the system to the average number of contiguous busy nodes starting from the first node of the network.

U-Statistics and the double stable integral

Joop Mijnheer, University of Leiden, The Netherlands

Multiple Wiener integrals appear in the theory of U-statistics. See Denker. Several results are known for multiple integrals with respect to a *symmetric* stable process (or measure). See Samorodnitsky and Taqqu and the references in this paper. We consider multiple integrals with respect to a *completely asymmetric* stable process. We compute the characteristic function (c.f.). The tail behaviour of the multiple integral will be derived from the c.f. The behaviour is different from the symmetric case.

References

M. Denker, Asymptotic Distribution Theory in Non-parametric Statistics, Advanced Lectures in Math. (Vieweg, Braunschweig/Wiesbaden).

G. Samorodnitsky and M.S. Taqqu, Multiple stable integrals of Banach-valued functions, Preprint.

Large deviation results for a class of Banach space valued Markov chains with application to population genetics

G.J. Morrow, University of Colorado at Colorado Springs, CO, USA

The infinitely many alleles model in population genetics gives rise to a probability measure valued Markov chain as follows. View an allele as a long sequence of nucleotide sites. The type t of an allele will be the site of its most recent mutation: $0 \leqslant t \leqslant 1$. Given a population of N alleles in generation n with empirical distribution $\nu = \nu_n = \sum \delta(t_i)/N$, we get a measure on $[0, 1]$ in generation $n+1$ defined by $\nu_{n+1} = \sum \delta(t_i')/N$ where (t_i') are i.i.d. with common distribution ν', with an appropriate transition mechanism $\nu \to \nu'$. We study the Markov Chain (ν_n) given by

$$\nu'(A) = (1-u)\int_A (1+s(t))\,\mathrm{d}t \Big/ \int (1+s(t))\,\mathrm{d}t + u|A|, \quad A \subset [0, 1],$$

for a mutation parameter u and a fitness parameter $s(t)$. Let these parameters admit the scaling $u = \mu\beta$ and $s(t) = \beta\sigma(t)$ where β gives the relative size of the rates of mutation and selection. It is assumed these evolutionary effects are strong: $N\beta \to \infty$. The transition $\nu \to \nu'$ has a stable fixed point with density $g_0(t) = u/[1-\gamma_0(1+s(t))]$ when we view $\nu = \int \varphi \, d\nu$, $\varphi \in \Lambda$, as an element of the Banach space $B = C(\Lambda)$, where Λ is the compact subset of $C[0, 1]$ consisting of Lipschitz functions with uniform and Lipschitz norms both bounded by 1. The logarithm of the expected time for the chain $\{\nu_n\}$ to leave an open ball D about g_0 in B is proved to be asymptotically $N\beta V$ for

$$V = \inf -2\left[\left[\int s(t)(p(t)-g_0(t))\,dt + \mu \int \log(p(t)/g_0(t))\,dt\right]\right],$$

where the infimum is extended over all densities p in the complement of D. The proof relies on an infinite dimensional extension of the method of G.J. Morrow and S. Sawyer appearing in "Large deviation results for a class of Markov chains arising in population genetics", Ann. Probab. (1989).

Asymptotics for point processes
Gert Nieuwenhuis, Katholieke Universiteit Brabant, The Netherlands

Let P be the distribution of a stationary point process on the real line and let P^0 be its Palm distribution. Our emphasis is on limit theorems, strong and weak (laws of large numbers, log log laws, marginal and functional central limit theorems). Two types are considered, those in terms of the number of points of the point process in $(0, t]$ and those in terms of the location of the nth point right of the origin. The former are most easily considered under P and the latter under P^0. General conditions are presented that guarantee equivalence of either type of limit theorem under both probability measures. As a bridge between P^0 and P we consider a third probability measure, P_1, arising from P by shifting the origin to the first point of the process on the right.

The obtained results for either type of limit theorem are extended to equivalances between the two types. Some remarks are made on generalisation to non-stationary point processes and marked point processes.

Loud shot noise
George L. O'Brien, York University, Downsview, Ont., Canada
(joint work with *R. Doney, University of Manchester, UK*)

Let $\{\tau_n\}_{-\infty<n<\infty}$ denote the renewal epochs of a stationary renewal process. We suppose a 'shot' occurs at each τ_n and contributes noise $h(t-\tau_n)$ at time t, where $h:[0,\infty)\to[0,\infty)$ is a fixed function. We assume $h(0)=1$ and h is nonincreasing. Then the total noise at time t is

$$\xi(t) = \sum_\tau h(t-\tau)$$

where τ runs through the renewal epochs up to t. Under modest conditions on h, $P[\xi(t) > x+1 \mid \xi(t) > x] \to 0$ as $x \to \infty$, for all t. Under more restrictive conditions on h, the distribution of $\max\{\xi(\tau_1), \ldots, \xi(\tau_n)\}$ behaves in the same way as $\max\{\eta_1, \ldots, \eta_n\}$, where the η_i's are i.i.d. with the same distribution as $\xi(\tau_1)$.

A problem of expectation and variance for a class of formal languages
Gabriel V. Orman, University of Braşov, Romania

One of the most interesting fields in which the probability theory can be applied is offered by the mathematical theory of formal languages. In the last years we have been concerned with the question of some new characterizations of the process of generation of the words by derivations according to formal grammars. In our studies the grammars constituting the so-called 'Chomsky hierarchy' have been considered. To this end we have defined some numerical functions able to characterize the derivations according to such a grammar up to an equivalence (that we call 'derivational functions').

In our paper "Two problems concerning the probability of generation of the words" (in print), we develop a particular study of Markov dependence in the process of generation of the words by derivations according to a phrase-structure grammar. Thus, if D_x is the equivalance class of a deviation $D(x)$, $x \geq 2$, we denote by ν_x the number of derivations of D_x by which a word w is generated. Evidently, w can or cannot be generated into D_x. If it is, then the probability that w should be generated into D_{x+1} is denoted by γ, if w is not generated into D_x, the probability that it should be generated into D_{x+1} is denoted by β. In this way we are in the case when the equivalence classes D_x, $x \geq 2$, are connected into a simple Markov chain. In this paper we continue this study involving one of the derivational functions namely the associated function to a derivation. By means of this function we determine both the probability that w should be generated into the class D_x and the probability that w should be generated into the class D_j if it was generated into the class D_i, $i<j$. But ν_x is a random variable that takes values 1 and 0 with probabilities p_x and $q_x = 1 - p_x$ respectively. We then determine the expectation and the variance of the random variable giving the number of derivations in n equivalence classes by which a word w is generated. Finally, we characterize the obtained language to the level of this function.

The size of domains of set-indexed Lévy processes

Mina Ossiander, Oregon State University, Corvallis, OR, USA

This paper examines the size of domains of set-indexed Lévy processes with independent increments. Members of this family of infinitely divisible random measures are independent on disjoint sets and have regular sample paths in the sense of outer continuity and inner limits. An integral condition involving the notion of majorizing measure is developed which limits the size of possible domains. This condition depends on both the Lévy measure of the process and the local complexity of the domain. This work extends results of Adler and Feigin (1984) and Bass and Pyke (1984). A counterexample of Adler and Feigin (1984) is generalized to show that for p-stable Lévy processes the integral condition is close to optimal.

On the tightness of the partial sums of a Φ-mixing sequence

Magda Peligrad, University of Cincinnati, OH, USA

We consider the partial sums of a stationary Φ-mixing sequence of centered L_2-integrable random variables having the property that $\Phi_1 < 1$, and we prove that a condition responsible for tightness always holds. As a consequence, if $S_n/\text{var}(S_n)$ converges to a nondegenerate distribution, the distribution is standard normal.

Precision bounds on the L_p norm of a quadratic form of independent mean zero random variables

Victor H. de la Peña, Columbia University, New York, USA*
Michael J. Klass, University of California, Berkeley, CA, USA

Let $X_1, X_2, \ldots, X_n$ be independent mean zero random variables, $S_n = X_1 + X_2 + \cdots + X_n$. Fix $p \geq 1$. Let $1 \leq J_n \leq n$ be any index for which

$$E|X_{J_n}|^p = \max_{1 \leq i \leq n} E|X_i|^p.$$

Then, there exist constants $0 < A_p, B_p < \infty$, such that,

$$A_p E|S_n|^p E|S_n - X_{J_n}|^p \leq E\left|\sum_{1 \leq i < j \leq n} X_i X_j\right|^p \leq B_p E|S_n|^p E|S_n - X_{J_n}|^p$$

where A_p, B_p do not depend on n or the distributions of the random variables.

On arrivals and departures that see the same average in bulk queues

David Perry and Michael J. Magazine, University of Waterloo, Ont., Canada*

We consider a general multi-server queueing system with batch arrivals in which the arrival process is a renewal process. For this general system we show that there are special relationships between the limiting probability law at departure times and the arrival times. For the special cases of Poisson arrivals or exponential service time, we generalize these relationships to the stationary distribution in time average.

Non-linear filtering with small noise coefficients

Jean Picard, INRIA - Sophia Antipolis, Valbonne, France

Consider the non-linear filtering problem where the signal X_t and the observation Y_t are solutions of

$$\mathrm{d}X_t = \beta(t, X_t)\,\mathrm{d}t + \sqrt{\varepsilon}\,\sigma(t, X_t)\,\mathrm{d}W_t + \sqrt{\varepsilon}\,\gamma(t, X_t)\,\mathrm{d}B_t,$$
$$\mathrm{d}Y_t = h(t, X_t)\,\mathrm{d}t + \sqrt{\varepsilon}\,\mathrm{d}B_t,$$

with two independent Brownian motions W_t and B_t. The aim of this work is to study asymptotically the conditional law of X given Y as $\varepsilon \to 0$. Firstly, we find conditions under which the variance of the conditional law if of order ε for sufficiently large times; these conditions include the case where $x \mapsto h(t, x)$ is one-to-one (the easiest case) but also some other situations; this problem is linked with the existence of non-linear observers for the associated deterministic system (obtained for $\varepsilon = 0$) and use the notion of detectability. Secondly, we check that under some additional assumptions, the conditional law is asymptotically Gaussian; in particular this provides a rigorous justification of the famous extended Kalman filter; this suboptimal filter, based on a linearization of the system, is proved nearly optimal in this case; however, we also describe other cases where it behaves badly.

By a change of time scale, we can also study the situation where the orders of magnitude of the signal and observation noises are different; with this remark, we can recover as a particular case the results of previous works (see references). The mathematical tools which are needed include the stability of stochastic systems, some estimates of large deviations, some changes of probability measures and the stochastic calculus of variations.

References

J. Picard, Nonlinear filtering of one-dimensional diffusions in the case of a high signal-to-noise ratio, SIAM J. Appl. Math. 46 (1986) 1098–1125.

J. Picard, Nonlinear filtering and smoothing with high signal-to-noise ratio, in: Stochastic Processes in Physics and Engineering (Bielefeld, 1986) (Reidel, Dordrecht, 1988).

Central limit and recurrent behavior of diffusions in a random environment

Ross Pinsky, Technion, Haifa, Israel
Mark Pinsky, Northwestern University, Evanston, IL, USA*

Let $(X(t), \sigma(t))$ be the Markov process on the state space $\mathbb{R}^d \times \{1, \ldots, N\}$ with the infinitesimal generator $L = \frac{1}{2}\Delta + G + V(x, \sigma)$ where G is the infinitesimal matrix of a Markov chain on $\{1, \ldots, N\}$, Δ is the Laplacian of $\mathbb{R}^d$ and $V(x, \sigma)$ is a homogeneous vector field with mean zero (w.r.t. the invariant measure of $\{0(t): t \geq 0\}$).

Theorem 1. *When $\varepsilon \downarrow 0$, $\varepsilon X(t/\varepsilon^2)$ converges weakly to a diffusion process on $\mathbb{R}^d$ with generator $L_0 = \frac{1}{2}\Delta + \sum_{\alpha\beta} c_{\alpha\beta} V(x, \alpha) V(x, \beta)$ where $c_{\alpha\beta}$ is the recurrent potential kernel of the Markov chain.*

Theorem 2. *If $d = 2$, the process $\{X(t): t \geq 0\}$ is recurrent.*

Theorem 3. *If $d = 3$, the process $\{X(t): t \geq 0\}$ is transient if* $\sup|V(x, \sigma)|$ *is sufficiently small.*

A probabilistic analysis of Lipschitz points for nondifferentiable Λ^* functions

J.M. Anderson and L.D. Pitt, University of Virginia, Charlottesville, VA, USA*

A continuous periodic function $f(x)$ is in Λ^* if

$$\sup_{h>0,x} (1/h)|f(x+h) + f(x-h) - 2f(x)| < \infty.$$

Λ^* functions may be nowhere differentiable, as in the case with the Weierstrass function

$$f(x) := \sum_k 2^{-n} \cos 2^n x,$$

but must satisfy Lipschitz conditions on a large set. For $f \in \Lambda^*$ we use the martingale of dyadic difference quotients

$$D_n f(x) := 2^n \sum_k [f((k+1)/2^n) - f(k/2^n)] \cdot 1_{I_{n,k}}(x),$$

where $I_{n,k} = [k/2^n, (k+1)/2^n)$, to analyze the sets

$$A(c) = \left\{ x: \limsup_{h \to 0} |(1/n)(f(x+h) - f(x))| \leq c \right\}$$

and

$$B(c) = \left\{ x: \limsup_{h \to 0} |1/h\sqrt{\log|1/h|}\{f(x+h) - f(x)\}| \leq c \right\}.$$

Upper and lower bounds for the size of the sets $A(c)$ and $B(c)$ are obtained, which in some cases give exact measure functions.

Estimation of reliability of stress-strength models based on sample paths

Nader Ebrahimi and T. Ramalingam, Northern Illinois University, DeKalb, IL, USA*

The stress and strength patterns of many stochastic systems are time dependent. Consequently, traditional non-dynamic models for stress and strength are inadequate for assessing the reliability of such systems. Furthermore, it is more effective to monitor systems over time and estimate the reliability or predict the time to failure of the system in advance of failure, than assuming that failure times are available for inference about the system.

In this paper, we propose a general stochastic model for stress and strength of systems and illustrate related inferences in some special cases including the Brownian Motion Model.

Light and heavy traffic limits for two finite capacity queueing systems

Martin I. Reiman, AT&T Bell Laboratories, Murray Hill, NJ, USA*
Burton Simon, University of Colorado at Denver, CO, USA

We consider two related queueing systems which have two classes of customers: infinite capacity (IC) and finite capacity (FC). Both customer classes have Poisson arrival processes. Upon their arrival, IC customers enter the main queue, which is served by a single server. There is a pool of N tokens associated with FC customers, and an FC customer needs a token to enter the main queue. Customers in the main queue are served in first-come-first-served order. Service time distributions are general and (possibly) different for each class. In Model 1, if no token is available an arriving FC customer waits in a secondary queue until a token becomes available. In Model 2, if an arriving FC customer finds no token available it leaves the system.

We determine the asymptotic behavior of this system in light and heavy traffic. Our light traffic asymptotics consist of the first $N+1$ derivatives of sojourn time moments with respect to the arrival rate, at an arrival rate of zero. The heavy traffic limit of IC customers is the same in both models: a one dimensional diffusion similar to a Bessel process. The heavy traffic limit of IC customers is the same in both models: a one dimensional diffusion similar to a Bessel process. The heavy traffic limit of FC customers in Model 1 is a one dimensional reflected Brownian motion. These asymptotics can be combined, by interpolating between them, to yield an approximation for all arrival rates.

Approximations for diffusion process boundary hitting times

Gareth Roberts, University of Nottingham, England

Methods for approximating distributions of boundary hitting times for diffusion processes will be considered. First, limit theorems for the distribution of a diffusion conditioned not to hit a time-invariant boundary are established, leading to tail estimates for the hitting time of a constant boundary. Secondly, these results are used to give asymptotically correct estimates of time dependent boundary hitting times.

Applications of these results in statistics will be briefly discussed.

Brownian motion in a wedge with variable skew reflection

L.C.G. Rogers, University of Cambridge, UK

In recent years, there has been considerable interest in the behavior of Brownian motion in a wedge with skew reflection on the two sides of the wedge, the direction of reflection being assumed constant on the two sides of the wedge. This talk will discuss the reachability of the vertex when the direction of reflection is allowed to vary along the edges. By techniques of complex analysis, it is possible to find a necessary condition, and a sufficient condition, for the vertex to be reachable. These conditions are of a geometric nature; the gap between them is evidently quite small!

A representation-independent integral for set-valued random variables

David Ross, University of Minnesota, Duluth, MN, USA

If χ is an infinite-dimensional Banach space, and F and F' are random subsets of χ (i.e., random variables taking values in $\mathcal{P}(\chi)$), then it can happen that F and F' have the same distribution but different integrals (in the sense of the Aumann integral). Debreu's notion of integration addresses this problem, at the cost of restricting the class of integrable sets.

In this talk I discuss several natural definitions for the integral of a general set-valued random variable which depend solely on the r.v.'s distribution. Remarkably, these definitions all produce the same integral.

Stochastic geometry and relativistic path integrals

Sisir Roy, Indian Statistical Institute, Calcutta, India

Feynman examined the Klein-Gordon and Dirac equations by the method of introducing a fifth coordinate u which played the role of a proper time. The jet extension of the Finslerian fields displays the idea of covariance of the physical field theory with respect to general changes of the parameter defined along the trajectories. It seems that this offers an attractive and quite obvious opportunity of consistent construction of dynamics of scalar, spinor, gauge etc, physical fields with respect to the parameter t. We propose to call and consider such parameter t as the evolution time parameter. A field can be said to be evolutionary-dynamic if it is given by a Lagrangian involving the first derivative of the field with respect to the evolutionary time parameter. This velocity can take arbitrary values at a certain point with fixed x^i and t in the four dimensional differentiable Finslerian Manifold. Then the metric tensor will depend on the random velocity variable. Now by using such covariant parametrization and arbitrary velocity vector, the description of Feynman's path as trajectories is possible even for an individual particle.

On the finiteness of the moments of first-crossing times from nonlinear renewal theory

Valeri T. Stefanov, Bulgarian Academy of Sciences, Sofia, Bulgaria, and Odense University, Denmark

The following stopping time covers a number of important special cases occurring in nonlinear renewal theory and sequential analysis:

$$\tau = \inf\{n \colon ng(n^{-1}Z_n) \geqslant a\}, \quad a > 0,$$

where Z_n is a random walk with a positive drift (μ) and $g(\cdot)$ is a positive function which is twice continuously differentiable in a neighborhood of μ. We show that all moments of τ are finite in the case when the random walk follows a distribution from any noncurved exponential family. This is also true for the continuous-time analogue of τ.

Product form in migration processes with batch movements

W. Henderson, University of Adelaide, S.A., Australia
P. Taylor, University of Western Australia, Nedlands, W.A., Australia*

Walrand (1983) and Pujolle (1988a, b) have recently derived the equilibrium colony size distribution for certain migration processes in which customers can arrive and depart from colonies in

batches. In this talk we will define a general class of migration processes which includes the models of both Walrand and Pujolle and discuss a method by which the equilibrium colony size distribution may be found.

It turns out that for processes satisfying appropriate conditions the equilibrium distribution has a form which is analogous to the product form which has been observed in migration processes with single customer movements by, among others, Jackson (1957), Baskett, Chandy, Muntz and Palacios (1975) and Kelly (1979).

References

F. Baskett, K. Chandy, R. Muntz and J. Palacios, Open, closed and mixed networks of queues with different classes of customers, J. Assoc. Comp. Mach. 22 (1975) 248-260.

J. Jackson Networks of waiting lines, Oper. Res. 5 (1957) 518-521.

F. Kelly, Reversibility and Stochastic Networks (Wiley, London, 1979).

G. Pujolle, On discrete time queueing systems, Preprint (1988a).

G. Pujolle, G. Multiclass discrete time queueing systems with a product form solution, Preprint (1988b).

J. Walrand, A discrete-time queueing network, J. Appl. Probab. 20 (1983) 903-909.

Sums and maxima of stationary sequences

K.F. Turkman, University of Lisbon, Portugal

Let $\{X_n\}$ be a stationary uniform mixing sequence with distribution function $F(\cdot)$ and mixing coefficient $\varphi(j)$ satisfying

$$\sum_{j=1}^{\infty} \varphi^{1/2}(j) < \infty. \tag{1}$$

Let $\{X_n^*\}$ be the associated i.i.d. sequence with the same distribution function. Let (a_n, b_n) and (c_n, d_n) be constants such that $a_n^{-1} \sum_{i=1}^{n} (X_i^* - b_n)$ and $c_n^{-1}(\max_{1 \le i \le n} X_i^* - d_n)$ converge in distribution to S and M respectively, where S is stable with index $0 < \alpha \le 2$ and M is a non-degenerate random variable. In this paper, under a local dependence condition and (1), the joint limiting distribution of

$$a_{i=1}^{-1} (X_i - b_n), \qquad c_n^{-1}\left(\max_{1 \le i \le n} X_i - d_n \right),$$

is studied and the possible limiting forms are given. It is shown that, except for the case when S is stable with index $\alpha < 2$ and M is max-stable with the same index, the normalized sums and maxima are independent.

References

C.W. Anderson and K.F. Turkman, The joint limiting distribution of sums and maxima of stationary sequences (1989), to be submitted.

T.L. Chow and J.L. Teugels, The sum and the maxima of i.i.d. random variables, in: Proc. 2nd Prague Symp. Asymptotic Statistics (1978) 81-92.

R.A. Davis, Stable limits for partial sums of dependent random variables, Ann. Probab. 11 (1983) 262-269.

Nonparametric inference for a doubly stochastic Poisson process

Klaus J. Utikal, University of Kentucky, Lexington, KY, USA

Consider a doubly stochastic Poisson process whose intensity λ_t is given by $\lambda_t = \alpha(Z_t)$, where α is an unknown nonrandom function of another (covariate) process Z_t. Only one continuous time-observation of counting and covariate process is available. The function $\mathscr{A}(z) = \int_0^z \alpha(x)\,dx$ is estimated and the normalized estimator is shown to converge weakly to a Gaussian process as time approaches infinity. Confidence bands for $\mathscr{A}$ are given. Tests for independence from the covariate process are proposed.

New results for optimal switching between a pair of Brownian motions

Robert J. Vanderbei, AT&T Bell Laboratories, Murray Hill, NJ, USA

We consider the problem to find the T^* that achieves the following sup

$$v(x) = \sup_T E_x f(B^1_{T_1(\tau)}, B^2_{T_2(\tau)}),$$

where $B(s) = (B^1_{s_1}, B^2_{s_2})$ is a two-parameter Brownian motion, f is a pau-off function defined on the boundary of the unit square in $\mathbb{R}^2$, τ is the first exit time from the unit square and $T = \{T(t) = (T_1(t), T_2(t)): 0 < t < \infty\}$ is an optional increasing path.

In this talk we quickly review old results and then discuss new ones.

This is joint work with Avi Mandelbaum and Larry Shepp.

Log log laws for geometric and full sequences

Wim Vervaat, Catholic University, Nijmegen, The Netherlands

Why are functional log log laws like Strassen's proved first for geometric subsequences? Is the subsequence result indeed weaker? The language of dynamical systems gives the best insight.

Let a group G act continuously on a metric space S. Let γ be a measurable function from $\mathbb{R}$ into G which is *regularly varying*, i.e., $\lim_{t\to\infty} \gamma(t+a)(\gamma(t))^{-1} = \delta(a)$ in G (consequently, $\delta(a+b) = \delta(a)\delta(b)$, $a \mapsto \delta(a)$ is continuous and the convergence is locally uniform). Let $f \in S$ be fixed and such that for each sequence $t_n \to \infty$ in $\mathbb{R}$ the sequence $\gamma(t_n)f$ in S has a convergent subsequence, and let K be the set of limit points in S of $\gamma(t)f$ as $t \to \infty$. For real $a > 0$, let $K_a \subset K$ be the set of limit points of $\gamma(na)f$ as $n \to \infty$, $n \in \mathbb{N}$.

Questions: when do we have $K_a = K$; for how many a can we have $K_a \neq K$? Here is the translation of Strassen's log log law for Brownian motion B into this language:

$$S = C[0, \infty),$$

$$f = B(\cdot, \omega) \quad \text{for a fixed sample point } \omega,$$

$$\gamma(\log t)f = \frac{f(t\cdot)}{\sqrt{2t \log\log t}},$$

$$\delta(\log a)f = a^{-1/2} f(a\cdot),$$

$$K = \left\{ f \in S: f(0) = 0, f \text{ absolutely continuous}, \int_0^\infty (f'(t))^2\,dt \leq 1 \right\}.$$

Several other functional log log laws to which this can be applied are being explored now.

Expected amount of overflow in a finite dam

P.R. Vittal, Western Carolina University, Cullowhee, NC, USA

In a finite dam model with random inputs the expected amount of overflow in a given time is analyzed using imbedding method. A second order differential equation for the expected amount of overflow is obtained and its complete solution is also derived. When the input is random and the output is proportional to the content of the dam, a second order differential equation of Kummer's type is obtained and its closed form solution is derived in terms of confluent hypergeometric functions. In addition to random outputs, if a deterministic exponential release policy is also followed we arrive at a third order differential equation for the expected amount of overflow and its solution is in terms of beta functions and degenerate hupergeometric functions of two variables.

Random walk approximations of skew and sticky Brownian motion

J.A.M. van der Weide, Delft University of Technology, The Netherlands

Let $(S_m)_{m\geq 0}$ be a Markov chain on $\mathbb{Z}$ which behaves outside 0 like symmetric random walk. From 0 the chain makes a transition to a state k according to a probability distribution $(p_k)_{k\in\mathbb{Z}}$ on $\mathbb{Z}$ with finite first moment. Consider the process $X_n = (X_n(t))_{t\geq 0}$, $n = 1, 2, \ldots$, defined by

$$X_n(t) = n^{-1/2} S_{[nt]}.$$

Harrison and Shepp (1981) claim weak convergence of the sequence X_n to skew Brownian motion with a parameter depending on the distribution $(p_k)_{k\in\mathbb{Z}}$. We will present a proof of this result using semigroup methods. As a special case of the more general problem where the probability distribution $(p_k)_{k\in\mathbb{Z}}$ depends on n, we will discuss a random walk approximation of sticky Brownian motion.

Reference

J.M. Harrison and L.A. Shepp, On skew Brownian motion, Ann. Probab. 9 (1981) 309-313.

The substitution method for bond percolation critical probability bounds

John C. Wierman, Johns Hopkins University, Baltimore, MD, USA

A 'substitution method' for deriving bounds for critical probabilities of bond percolation models will be presented. In this method, a lattice is compared with another lattice for which the critical probability is exactly known. Portions of one lattice are substituted into the other, while an appropriate transformation is applied to the parameters in the bond percolation model. The transformation is determined from a stochastic ordering on probability measures derived from finite random graphs corresponding to portions of the lattices. The method theoretically provides sequences of upper and lower bounds for the bond percolation critical probability. Both upper and lower bounds for the bond percolation critical probability of the Kagome lattice are substantially improved by an application of this method.

On the quasireversibility of a multiclass Brownian service station

J.M. Harrison, Stanford University, CA, USA
R.J. Williams, University of California at San Diego, CA, USA*

The object of study in this paper is a Brownian model of a multiclass service station. Such Brownian models arise as heavy traffic limits of conventional queueing models in which several different types or classes of customers are processed through a common service facility. Assuming that the Brownian service station is initialized with its stationary distribution, four different model characteristics are shown to be equivalent, and the station is said to be quasireversible if those equivalent conditions pertain. Three of the four conditions characterize the vector output process from the Brownian service station, and our definition of quasireversibility parallels precisely that proposed by F.P. Kelly for conventional queueing models. The last of our four conditions is expressed directly in terms of primitive station parameters, so one may easily determine from basic data whether or not a Brownian station model is quasireversible. In future work we hope to show that a network of quasireversible Brownian service stations has a product form stationary distribution.

Equivalent martingale measures and no-arbitrage in stochastic securities market models

Robert C. Dalang, University of California at Berkeley, CA, USA
Andrew Morton, University of Illinois at Chicago, IL, USA
Walter Willinger, Bellcore Morristown, NJ, USA*

We characterize those vector-valued stochastic processes (with a finite index set and defined on an arbitrary stochastic base) which can become a martingale under an equivalent change of measure.

This question is important in a widely studied problem which arises in the theory of finite period securities markets with one riskless bond and a finite number of risky stocks. In this setting, our characterization gives a criterion for recognizing when a securities market model allows for no arbitrage opportunities ('free lunches'). Intuitively, this can be interpreted as say "if one cannot win betting on a process, then it must be a martingale under an equivalent measure", and provides a converse to the classical notion that "one cannot win betting on a martingale".

Some new Vapnik - Chervonenkis classes of sets

G. Stengle and J.E. Yukich, Lehigh University, Bethlehem, PA, USA*

This paper provides some new examples of Vapnik - Chervonenkis (VC) classes of sets and shows that these classes fit naturally within the framework of algebraic geometry. In particular, semialgebraic sets are linked to VC classes and are used to manufacture relatively large VC families of positivity sets, thus extending a result of Dudley (Ann. Probab. 6, 1978, 899-929). The main tool here is the quantifier elimination theorem of Tarski-Seidenberg. Similarly, tools from the theory of analytic functions are used to show that certain special analytic families of positivity sets are VC.